Minimum Energy Multicast Routing for Wireless Ad-hoc Networks with Adaptive Antennas

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Abstract

Energy conservation is a critical issue in wireless multihop ad-hoc networks, which have nodes powered by batteries only. One major metric for energy conservation is to route a communication session along the routes that require the lowest total energy consumption. In this paper, we consider wireless ad hoc networks that use adaptive antennas and have limited energy resources. To explore the advantages of power saving offered by the use of adaptive antennas, we consider the case of source initiated multicast traffic. We present a constraint formulation for the MEM (Minimum-Energy Multicast) problem in terms of MILP (Mixed Integer Linear Programming) for wireless ad hoc networks. Experiment results show that an optimal solution of the MEM problem using our MILP model can always be obtained in a timely manner for moderately sized network, and it also provides a way to evaluate the realistic performance of different heuristic algorithms.

1. Introduction

An ad hoc network is a peer-to-peer mobile network consisting of large number of mobile nodes. These nodes create an instant network on demand and may communicate with each other via intermediate nodes in a multi-hop mode, i.e., every node can be a router. Ad hoc networks may be the only solution in many situations where instant infrastructure is needed and no central backbone system and administration (like base stations and wired backbone in a cellular system) exist. Some of the applications include mobile computing in areas where other infrastructure is unavailable, law enforcement operations, as well as disaster recovery situations. Each node in such a network has a limited energy resource (battery), and each node operates in an unattended manner. Consequently, energy efficiency is an important design consideration for these networks. In this paper, we explore the energy conservation offered by the use of directional antennas for broadcasting /multicasting in wireless ad hoc networks.

The broadcast/multicast communication is an important mechanism to communicate information in wireless ad hoc networks. This is because the network described above can be regarded as a distributed system, where broadcast /multicast is an important communication primitive. In addition, many routing protocols for wireless ad-hoc networks need a broadcast /multicast mechanism to update their states and maintain the routes between nodes.

When power efficiency is considered, ad hoc networks will require a power-aware metric for their routing algorithms. Typically, there are two main optimization metrics for energy-efficiency broadcast/ multicast routing in wireless ad hoc networks:

(1) Maximizing the network lifetime; and
(2) Minimizing the total transmission power assigned to all nodes.

Maximum lifetime broadcast/multicast routing algorithms [24, 28, 29, 30] can distribute packet-relaying loads for each node in a manner that prevents nodes from being overused or abused. By maximizing the lifetime of all nodes, the time before the network is partitioned is prolonged. A lot of work for the broadcast /multicast [17, 18, 20, 21, 22] using the minimum total transmission power as optimization metric is based on the obvious intuition that conserving power will ensure the network lifetime to be increased. Most recent work has been proposed for the problems of minimizing the energy consumption for broadcasting and multicasting in wireless ad hoc networks, addressed as the MEB (Minimum-Energy Broadcast) problem and MEM (Minimum-Energy Multicast) problem respectively.

Since both the MEB problem and the MEM problem, a special case of MEM, have recently been shown to be NP-hard [23, 25], efficient heuristic algorithm design

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has received much more attention [17, 18, 20, 21, 22]. While the performances of these algorithms can certainly be compared among themselves, in the absence of any optimal solution, it has not been possible to judge the quality of the solutions with respect to the optimal.

This paper attempts to fill that void by proposing a general analytical MILP (Mixed Integer Linear Programming) model for the MEM problem in an ad-hoc network equipped with adaptive antennas. Thus the MEM problem can be solved by any standard linear programming based branch-and-bound technique. This model discussed in this paper assumes global knowledge of pair-wise distances between the nodes and is therefore most suited for static networks. Our simulation results show that an optimal solution of the MEM problem using our model can always be obtained in a timely manner for moderate networks typically with 50 nodes.

The remaining of this paper is organized as follows. In Section 2, we overview related work concerning using directional antennas in ad hoc wireless networks and minimum-energy broadcast/multicast problem. In Section 3, we present the adaptive antenna propagation model. In Section 4, we give a definition of minimum energy multicast tree in the context of directional antenna applications. Section 5 gives the linear constraints for Problem MEM systematically, and completes the formulation of the problem in a form of Mixed Integer Linear Programming. Computational results assessing the performance using several algorithms for many network examples are in Section 6. Finally, we summarize our finding and points out several future research problems in Section 7.

2. Related Work

2.1. Directional Antennas

It has been shown earlier that the use of directional antenna in the context of ad hoc wireless networks can largely reduce the radio interference, thereby improving the utilization of wireless medium and consequently the network performance [1, 2, 3, 4, 5]. Some papers [8, 9] suggest the use of multiple directional antennas per node (or multiple beam antennas), in order to increase the throughput of 802.11 media access control protocol [10]. In [11] the author explores the use of beam forming antennas in order to improve both throughput and delay in ad-hoc networks. Another paper [12] has suggested the use of multiple directional antennas to reduce the routing overhead of on-demand routing protocols for ad-hoc networks like DSR (Dynamic Source Routing) [13] and AODV (Ad-hoc On Demand Distance Vector) [14].

Over the last few years, energy efficient communication in wireless ad hoc networks with directional antennas has received more and more attention, since one important characteristic of such networks is that nodes are energy-constrained. Nodes are battery-operated and frequent recharging or replacement of batteries may be undesirable or even impossible. In [6, 7], the authors presented a power-efficient algorithm called S-GPBE (Sectored Greedy Perimeter Broadcast Efficiency) exploiting broadcast efficiency for wireless ad hoc networks using directional or sector antennas. An energy-efficient routing and scheduling algorithm [15] was used to coordinate transmissions in ad hoc networks where each node has a single directional antenna.

2.2. MEB/MEM Using Omni-Directional Antennas

For the MEB problem, a straight greedy approach is the use of broadcast trees that consist of the best unicast paths to each individual destination from the source node (broadcast session initiator). This heuristic first applies the Dijkstra's algorithm to obtain a SPT (Shortest Path Tree), and then to orient it as a tree rooted at the source node. Similarly the MST (Minimum Spanning Tree) heuristic first applies the Prim's algorithm to obtain a MST, and then to orient it as a tree rooted at the source node.

In [17, 18], another heuristic algorithm for the MEB problem called BIP (Broadcast Incremental Power) was presented. The BIP algorithm is similar in principle to the standard Prim algorithm for the formation of minimum spanning trees. It maintains throughout its execution a single tree rooted at the source node. Initially, the rooted tree only includes the source node. Subsequently the tree node that can cover a new node outside the rooted tree with the least incremental power expands its power range to include this new node in the rooted tree. This operation is repeated until all nodes are included in the tree. BIP exploits the wireless advantage property[1] in the formation of the broadcast trees, and thus provides better performance than the greedy algorithms SPT and MST. All the algorithms mentioned above are centralized. Recently, distributed algorithms RBOP (Related Neighbourhood Graph based Broadcast Oriented Protocol) [21] and EWMA (Embedded Wireless Multicast Advantage) [25] are shown to have comparable performance to BIP. In literature, the MEM problem was studied in a same approach as the MEB problem except that the final minimum-energy

[1]It means that all nodes within communication range of a transmitting node can receive a multicast message with only one transmission if they all use omni-directional antennas.

multicast tree is obtained by pruning from the minimum-energy broadcast tree all transmissions that are not needed to reach the member of the multicast group. When applied to the multicast problem, the resulting scheme of BIP is called MIP (Multicast Incremental Power) [17, 18].

2.3. MEB/MEM Using Directional Antennas

Wieselthier et al [16] first studied the MEB/MEM problems considering these two aspects simultaneously: energy conservation offered by use of directional antennas and the wireless advantage property for broadcasting /multicasting. The incremental power philosophy in BIP/MIP, originally developed for use with omni-directional antennas, can be applied to broadcast/multicast tree construction in networks with directional antennas as well. Two heuristic algorithms called RB-BIP/RB-MIP (Reduced Beam BIP / Reduced Beam MIP) and D-BIP/D-MIP (Directional BIP / Directional MIP) were then proposed as variant extensions of the BIP/MIP algorithm for the situation of using adaptive antennas. RB-BIP/RB-MIP [16, 19] algorithm is essentially same as BIP/MIP except that, after the BIP/MIP tree is constructed, the beamwidth of antenna is reduced to fit minimum possible angle to cover all child nodes of each node. D-BIP/D-MIP [16, 19] algorithm, another variant extension of BIP/MIP, utilizes wireless advantage property in the core of the algorithm while building a routing tree. At each step of the tree-construction process, a single node is added, as in BIP/MIP algorithm. However, whereas the only variable involved in computing the incremental power in the omni-directional case was the transmission range, the directional-antenna case involves the choice of antenna orientation and beamwidth as well.

On the other hand, both RB-BIP/RB-MIP and D-BIP/D-MIP inherit the disadvantages of BIP/MIP: for some instances the energy conservation nature of the directional antenna and the wireless advantage property of the media are ignored. This happens because they add just one node at each iteration step of the tree construction, the one that can be added at minimum additional cost, but do not use all available information about the network.

3. Antenna Model

In an ad-hoc wireless network each node is equipped with adaptive array antennas, which permits energy savings by concentrating transmission energy where it is needed. Adaptive array antennas are a set of antenna elements arranged in space whose outputs are combined to give an overall antenna pattern that can differ from the pattern of the individual elements. By varying the phase and amplitude of the individual element outputs before combining, the overall array pattern can be steered in the desired user's direction without physically moving any of the individual elements.

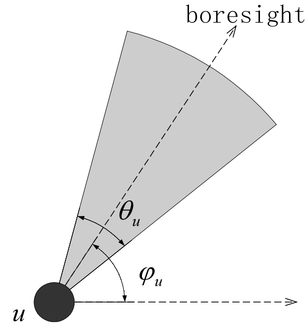


Figure 1. Directional antenna propagation model

We use an idealized adaptive antenna propagation model as shown in Fig 1, where the antenna orientation φ_v ($0 \le \varphi_v < 2\pi$) of node v is defined as the angle measured counter-clockwise from the horizontal axis to the antenna boresight, and the antenna directionality is specified as the angle of beamwidth θ_v ($\theta_v^{\min} \le \theta_v \le \theta_v^{\max}$). For our antenna propagation model, we assume that for any node v, all of the transmitted energy is concentrated uniformly in a beamwidth, ignoring the possibility of sidelobe interference.

Based on this model, the transmitted power required to support a link between two nodes separated by range r ($r > 1$) is proportional to r^α and beamwidth θ_v, where the propagation loss exponent α typically takes on a value between 2 and 4. Without loss of generality, all receivers have the same power threshold for signal detection, which are typically normalized to one, resulting in that the transmission power needed by node v to reach node u in its antenna beam coverage using beamwidth θ_v is

$$p_{uv} = r_{uv}^{\alpha} \cdot \theta_u / 2\pi \qquad (1)$$

where r_{vu} is the distance between node v and node u, and p_{vu} represents the power needed for link between node v and node u.

Consequently, the use of narrow beams permits energy saving for a given communication range or range extension for a given transmission power level as compared to the use of omni-directional antennas. Within the antenna beam of node v, nodes that are closer to v than u will also receive the transmission directed to u. Therefore, it is important to note how this wireless advantage property can be exploited in broadcast and multicast applications.

4. Minimum Energy Multicast Tree

Let us model the network by a simple directed graph $G(N, A, p)$, where N is a finite node set, $|N| = n$, and A is an arc set corresponding to the unidirectional wireless communication links. The arc weight function p: $A \rightarrow R^+$ assigns power to each arc, where R^+ denotes the positive real number set. That is, for each arc (v, u), p_{vu} is the power needed for the link from node v to node u. We assume that any node $v \in N$ can choose its power level, not to exceed some maximum value p^v_{max}.

We consider a source-initiated multicast in wireless ad-hoc networks. Any node is permitted to initiate multicast sessions. Multicast requests and session durations are generated randomly at the network nodes. The set of nodes M that support a multicast session, including the source node and all destination nodes, is referred to as a multicast tree. Any multicast tree is a rooted tree. We define a rooted tree as a directed acyclic graph with a source node called root with no incoming arcs, and all its other nodes with exactly one incoming arc. A property of rooted tree is that for any node u in the tree, there exists a single directed path from s to u in the tree. A node with no out-coming arcs is called a leaf node, and all other nodes are internal nodes, or relay nodes, whose antenna beams should cover all their children. The minimum-energy multicast problem is to find a multicast tree with the minimum power consumption. Doing so involves the choice of transmission power level, relay nodes, antenna beamwidth, and antenna orientation. The relay nodes may be multicast members or may not. Formally, a multicast tree is modeled by a node-weighted tree $T_s(N', A', q)$ rooted at a source node s, $s \in N$, with a multicast node set $N' \subseteq N$, an arc set $A' \subseteq A$, and a node weight function defined as q: $N' \rightarrow R^+ \cup \{0\}$. That is, for each node v in N', q_v is the transmission power of the node v required by the multicast tree T_s. We define $T_s(N', A', q)$ to be a multicast tree of $G(N, A)$ rooted at s if and only if the following properties are satisfied.

1) **RTP** (Rooted Tree Property) requires T_s can span all the multicast members from node s;
2) **WAP** (Wireless Advantage Property) requires the node weight function to satisfy:

$$q_v = \begin{cases} 0, & v \text{ is leaf node;} \\ \text{Max}\{p_{vu} \mid (v,u) \in A'\}, & v \text{ is internal node.} \end{cases} \quad (2)$$

3) **ACP** (Antenna Coverage Property) requires node u must be located within the antenna beam of node v, for any $(v, u) \in A'$.

We assume that no power expenditure is involved in signal reception and processing activities. Thus the total power is expended completely on transmission at each node in the tree. Obviously, leaf nodes do not contribute to this quantity because they do not relay traffic to any other nodes. Hence, we evaluate performance in terms of total power from all transmitting nodes required to maintain the tree.

5. MILP Model for MEM Problem

The definition of multicast tree given in the context of directional antenna applications allows us to formulate the MEM Problem as a MILP (Mixed Integer Linear Programming) model. The main idea is to extract a sub-graph T_s^* from the original graph G, such that T_s^* is a multicast tree rooted at node s with minimum energy consumption. In order to formulate the problem, we define the following variables:

(i) Z_{vu} is a binary decision variable which is equal to one if the arc (v, u) is in the sub-graph T_s^* of G, and zero otherwise;
(ii) P_v is a nonnegative continuous variable which represents the transmission power of the node v required by the multicast tree T_s^*;
(iii) F_{vu} is a nonnegative continuous variable that only represents fictitious flow produced by the multicast initiator s going through arc (v, u), and thus helps prevent loops.

We shall prove that if $(x)^*$ is the optimal solution of variable x obtained from this MILP model, then the graph $T_s^*(N', A', q)$ is the optimal tree associated with this solution, where $N' = \{u \mid \exists\, (v, u) \in A' \text{ or } (u, v) \in A'\}$ is its node set, $A' = \{(v, u) \mid Z^*_{vu} = 1\}$ is its arc set, and q: $N' \rightarrow R^+ \cup \{0\}$ is a nonnegative weight function defined as $q_v = P^*_v$. In other words, $T_s^*(N', A', q)$ is a multicast tree of G with minimum energy consumption.

5.1. Linear Constraints for RTP

We want to provide a set of constraints that would guarantee that $T_s^*(N', A', q)$ obtained from the formulation satisfies the rooted tree property. It can be characterized that $T_s^*(N', A', q)$ is a rooted tree spanning all the multicast members, i.e., $M \subseteq N'$, by the following properties. Theorem 1 can achieve these two properties, and the construction and interpretation of the linear constraints are elaborated in the proof.

RTP (a): Every node $u \in N' \setminus \{s\}$, has exactly one incoming arc, and node s has no incoming arcs;

RTP (b): $T_s^*(N', A', q)$ does not contain cycles.

Theorem 1. $T_s^*(N', A', q)$ is a rooted tree at node s, provided Problem MEM satisfies the following constraints:

$$\sum_{v\in N} Z_{vs} = 0; \quad (3)$$

$$\sum_{v\in N} Z_{vu} = 1;\ \forall u \in M\backslash\{s\} \quad (4)$$

$$\sum_{v\in N} Z_{vu} \le 1;\ \forall u \in N\backslash M \quad (5)$$

$$\sum_{v\in N} Z_{uv} \le (n-1) \sum_{v\in N} Z_{vu};\ \forall u \in N\backslash M \quad (6)$$

$$\sum_{v\in N} F_{vu} - \sum_{v\in N} F_{uv} = \sum_{v\in N} Z_{vu};\ \forall u \in N\backslash\{s\} \quad (7)$$

$$Z_{vu} \le F_{vu} \le (n-1)\, Z_{vu};\ \forall u \in N\backslash\{s\},\ v \in N \quad (8)$$

Proof: We first prove the RTP (a) case. Note that $\Sigma_{v\in N} Z^*_{vu}$ and $\Sigma_{v\in N} Z^*_{uv}$ are the in-degree and out-degree of node u in T_s^* respectively. Therefore, the root node s and the other multicast members satisfy RTP (a) directly from the Constraints (3) and (4) respectively. It remains to prove that any non-multicast member in T_s^* supporting the multicast communications must have exactly one incoming arc. Assume $u \in N'$ is a non-multicast member in T_s^*, indicated by a hollow node in Figure 2, its incoming degree must be 1 or 0 from Constraints (5). If $\Sigma_{v\in N} Z^*_{vu} = 0$, from Constraints (6), it follows that $\Sigma_{v\in N} Z^*_{uv} = 0$. That means u must be an isolated node as shown in Figure 2 a), thus $u \notin N'$. This contradicts the original assumption. Therefore node u has exactly one incoming arc.

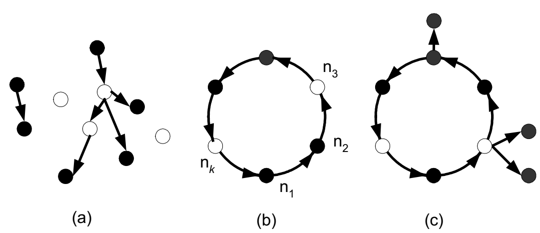


Figure 2. Illustration of constraints: (a) any non-multicast member in Ts* must have exactly one incoming arc, (b) a connected component of Ts* may be a simple cycle, (c) a cycle with sub tree leaving out of it. (Solid nodes indicate multicast members, and hollow nodes indicate non-multicast members.)

For the RTP (b) case, from the Constraints (3), (4) and (5), it follows that the only connected components in T_s^* that might contain cycles could be composed of either a simple cycle as shown in Figure 2 b), or a simple cycle with sub tree leaving out of it as shown in Figure 2 c). We will show in the following that such topologies are not feasible for Problem MEM. Assume that the nodes (n_1, n_2, …, n_k, $n_{k+1} = n_1$), $k > 1$, form a simple cycle in T_s^*. Then from Constraint (3), node s will never be included in such a cycle. Constraint in (8) implies that F^*_{vu} could be positive if and only if $(v, u) \in A'$. Letting $F^*_{n_1 n_2}$ be a constant f, then from the Constraint (7) it follows that $F^*_{n_r n_{r+1}} = F^*_{n_1 n_2} - \sum_{i=1}^{r-1} Z^*_{n_i n_{i+1}}$ for $r = 1, …, k$. Each node n_r ($r = 1, …, k$) is in A' as stated in the assumption above, i.e., $Z^*_{n_r n_{r+1}} = 1$. Therefore $F^*_{n_r n_{r+1}} = F^*_{n_1 n_2} - \sum_{i=1}^{r-1} Z^*_{n_i n_{i+1}} = f - (r-1)$ for $r = 1, …, k$. After substituting $F^*_{n_k n_1} = f - (k-1)$ into Constraint (7), for $u = n_1$, we obtain $\sum_{v\in N} F^*_{vn_1} - \sum_{v\in N} F^*_{n_1 v} = f - (k-1) - f = 1 - k < 0$. On the other hand, $\sum_{v\in N} F^*_{vn_1} - \sum_{v\in N} F^*_{n_1 v} = \sum_{v\in N} Z^*_{vn_1} \ge 0$. Thus the Constraint (7) is violated, and therefore simple cycles are not possible in T_s^*. Similar reasoning shows that the topology in Figure 2 c) also violates the Constraints (7), and therefore T_s^* cannot contain cycles. ■

5.2. Linear Constraints for WAP

The constraints for the wireless advantage property (WAP) reflect the condition that the power required at node u is the maximum of the individual transmission power to each neighbour from u. The Constraint (9) guarantees the WAP, i.e., Equation (2), can be easily achieved.

$$P_v \ge p_{vu} Z_{vu};\ \forall v, u \in N \quad (9)$$

This can be explained as follows. For any node v in T_s^*, if v is a leaf node, i.e., $Z^*_{vu} = 0$ for all $u \in N'$, then $P^*_v \ge p_{vu} Z^*_{vu} = 0$; if v is an internal node, then $P^*_v \ge p_{vu} Z^*_{vu}$ for all $u \in N'$, i.e., $P^*_v \ge Max_{(v,u)\in A'}\, p_{vu}$. The equalities are achieved in the inequations above when the summation of the variables P_v is minimized. Thus Equation (2) must be held by T_s^*. However, we also note that after substituting Equation (1) into Constraint (9), the multiplication form in $P_v \ge r^{\alpha}_{vu} \cdot Z_{vu} \cdot \theta_v / 2\pi$ appears nonlinear obviously. The following theorem illustrates how this constraint can be linearized.

Theorem 2. $T_s^*(N', A', q)$ satisfies WAP, if the formulation of Problem MEM includes the following constraints.

$$p^v_{max} \ge P_v \ge r^{\alpha}_{uv} \cdot Y_{vu} / 2\pi;\ \forall v, u \in N \quad (10)$$

$$0 \le Y_{vu} \le 2\pi Z_{vu};\ \forall v, u \in N \quad (11)$$

$$Y_{vu} \le \theta_v;\ \forall v, u \in N \quad (12)$$

$$\theta_v - Y_{vu} + 2\pi Z_{vu} \le 2\pi;\ \forall v, u \in N \quad (13)$$

Proof: In order to prove Constraints (10) to (13) are equivalent to Constraint (9), we only need to verify that variable Y_{vu} must satisfy the following conditions:

$$Y_{vu} = \theta_v Z_{vu} = \begin{cases} 0, & Z_{vu} = 0 \\ \theta_v, & Z_{vu} = 1 \end{cases} \quad (14)$$

When $Z_{vu} = 0$, from Constraint (11) $0 \le Y_{vu} \le 2\pi Z_{vu} = 0$ we have $Y_{vu} = 0$, and Constraints (12) and (13) become trivially true. When $Z_{vu} = 1$, Constraint (13) is simplified as $\theta_v \le Y_{vu}$, from which we conclude that $Y_{vu} = \theta_v$, considering Constraint (12) $Y_{vu} \le \theta_v$ at the same time.■

5.3. Linear Constraints for ACP

Before discussing the construction of linear constraints for ACP, we first investigate in more detail the relationship among the variables: antenna orientation φ_v, beamwidth θ_v, and Z_{vu} indicating if the arc (v, u) exists in the multicast tree T_s^*.

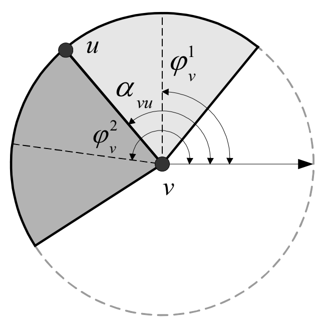


Figure 3. Antenna beam coverage range

Let α_{vu} ($0 \le \alpha_{vu} < 2\pi$) be the angle measured counter-clockwise from the horizontal axis to the vector $\overrightarrow{vu}$ as shown in Fig. 3. Then the angle α_{vu} ($v, u \in N$) can be obtained once their positions are given. In Fig. 3, the lighter shaded area is the space covered by the antenna beam of node v when it is about entering the position of node u (i.e., for v making contact with u), and the darker shaded area is the space just before the beam is leaving the position of node u (i.e., for v losing contact with u). Thus it is clear that the wireless link (v, u) exists in the multicast tree T_s^*, i.e., $Z_{vu} = 1$, only if the antenna orientation φ_v is bounded by the two pointing directions $\varphi^1_v=\alpha_{vu}-\theta_v/2$ and $\varphi^2_v=\alpha_{vu}+\theta_v/2$, indicated by the dotted lines as shown in Fig. 3.

In order to simplify our analysis, we first extend the constraint $\theta_v^{\min} \le \theta_v \le \theta_v^{\max}$ into $0 \le \theta_v \le 2\pi$. In a φ_v–θ_v plane as shown in Fig. 4a, the points (φ_v, θ_v) that satisfy the constraint $\alpha_{vu}-\theta_v/2 \le \varphi_v \le \alpha_{vu}+\theta_v/2$ must be within the area bounded by line AB, line AC, and line BC, where $A = (0, \alpha_{vu})$, $B = (2\pi, \alpha_{vu}-\pi)$, and $C = (2\pi, \alpha_{vu}+\pi)$. When we consider $0 \le \alpha_{vu} \le \pi$ and $0 \le \varphi_v < 2\pi$, this area must be mapped into the shaded area in Fig. 4a since φ_v and $\varphi_v+2\pi$ denote the same physical direction. Let I denote the area covered by triangle CFG, and II the area covered by quadrangle $ADEC$, where $D = (2\alpha_{vu}, 0)$, $E= (2\pi, 0)$, $F = (2\pi, 2\pi)$, and $G = (2\alpha_{vu}, 2\pi)$. Recall that ACP requires node u to be located within the antenna beam of node v, for any (v, u) included in the multicast tree T_s^*. Based on our analysis above, this property can be rewritten as $Z_{vu} = 1$ only if $(\varphi_v, \theta_v) \in \mathrm{I} \cup \mathrm{II}$. Since I and II are disjoint sets, Z_{vu} can be decomposed into a summation of two new binary variables Z_{vu1} and Z_{vu2}, where $Z_{vu1} = 1$ only if $(\varphi_v, \theta_v) \in \mathrm{I}$, and $Z_{vu2} = 1$ only if $(\varphi_v, \theta_v) \in \mathrm{II}$. Theorem 3 explains how property ACP can be satisfied by a set of linear constraints.

Theorem 3. For any (v, u) included in the multicast tree T_s^* and $0 \le \alpha_{vu} \le \pi$, node u must locate within the antenna beam of node v if the following constraints hold.

$$2\varphi_v + \theta_v - (4\pi + 2\alpha_{vu})Z_{vu1} \ge 0 \quad (15)$$

$$2\varphi_v - \theta_v + (4\pi - 2\alpha_{vu})Z_{vu2} \le 4\pi \quad (16)$$

$$2\varphi_v + \theta_v - 2\alpha_{vu}Z_{vu2} \ge 0 \quad (17)$$

Proof: We only need to prove that the statement "$Z_{vu} = 1$ only if $(\varphi_v, \theta_v) \in \mathrm{I} \cup \mathrm{II}$" is equivalent to the Constraints (15) to (17). Since $Z_{vu} = Z_{vu1} + Z_{vu2}$, and they are all binary variables, $Z_{vu} = 1$ if and only if just one of the Boolean expressions ($Z_{vu1} = 1$ and $Z_{vu2} = 0$) and ($Z_{vu1} = 0$ and $Z_{vu2} = 1$) is true. We first consider the case $Z_{vu1} = 1$ and $Z_{vu2} = 0$. Thus Constraints (15) – (17) become $2\varphi_v \ge \theta_v/2 + 2\pi + \alpha_{vu}$, $\varphi_v \le \theta_v/2 + 2\pi$, and $\varphi_v \ge -\theta_v/2$ respectively. Considering the boundary conditions $0 \le \theta_v \le 2\pi$ and $0 \le \varphi_v < 2\pi$, we observe that these constraints just define the area I as shown in Fig 4a. Similarly, after substituting $Z_{vu1} = 0$ and $Z_{vu2} = 1$ into Constraints (15) – (17), we can easily examine that the resulting constraints $\varphi_v \ge -\theta_v/2$, $\varphi_v \le \alpha_{vu} + \theta_v/2$, and $\varphi_v \ge \alpha_{vu} - \theta_v/2$ define the area II. Combing the two cases, we conclude that Constraints (15) – (17) characterize the statement "$Z_{vu} = 1$ only if $(\varphi_v, \theta_v) \in \mathrm{I} \cup \mathrm{II}$" correctly. ■

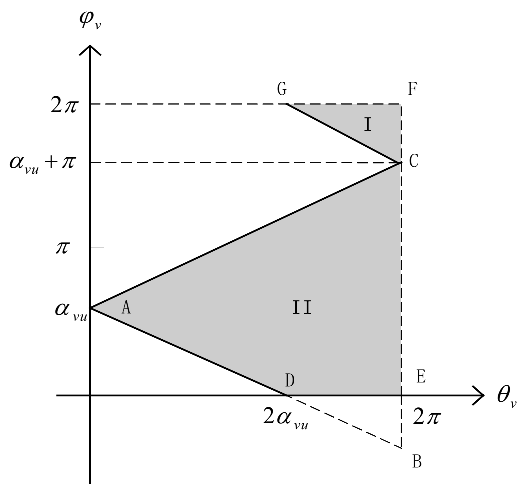
So far, we only consider the case $0 \le \alpha_{vu} \le \pi$. A similar constraint construction for property ACP can be made under condition $\pi < \alpha_{vu} < 2\pi$. Fig. 4b shows the shaded area, only in which the value of Z_{vu} can be equal to 1. The corresponding linear constraints that characterize property ACP for $\pi < \alpha_{vu} < 2\pi$ are summarized in the theorem below.

Theorem 4. For any (v, u) included in the multicast tree T_s^* and $\pi \le \alpha_{vu} \le 2\pi$, node u must locate within the antenna beam of node v if the following constraints hold.

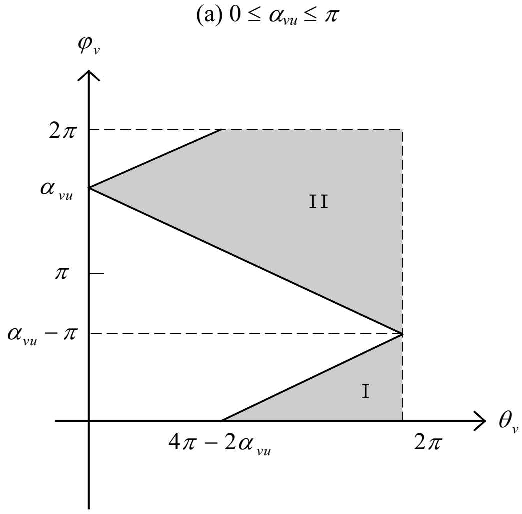
$$2\varphi_v - \theta_v + (8\pi - 2\alpha_{vu})Z_{vu1} \le 4\pi \tag{18}$$

$$2\varphi_v - \theta_v + (4\pi - 2\alpha_{vu})Z_{vu2} \le 4\pi \tag{19}$$

$$2\varphi_v + \theta_v - 2\alpha_{vu}Z_{vu2} \ge 0 \tag{20}$$



(a) $0 \le \alpha_{vu} \le \pi$



(b) $\pi < \alpha_{vu} < 2\pi$

Figure 4. Illustration of linear constraint construction for property ACP

5.4. Problem Formulation

Our previous derivation on the linear constraints can now help us to rewrite the problem formulation at the beginning of this Section as a MILP model. This is shown in Fig. 5, in which the coefficients A_{vu}, B_{vu}, and C_{vu} are given in Table 1.

Table 1. Value of coefficients

	$0 \le \alpha_{vu} < \pi$	$\pi \le \alpha_{vu} < 2\pi$
A_{vu}	-2	2
B_{vu}	$4\pi + 2\alpha_{vu}$	$8\pi - 2\alpha_{vu}$
C_{vu}	0	4π

In this formulation, Z_{vu1} and Z_{vu2} are binary variables; P_v, F_{vu}, θ_v, and φ_v are continuous variables. The number of variables in the formulation is approximately $3n^2 + 3n$, and the number of constraints is of the order of $O(n^2)$.

$$\text{minimize } \textstyle\sum_{u\in N} P_u \tag{21}$$

Subject to:

Rooted Tree Property

$\sum_{v\in N} (Z_{vs1} + Z_{vs2}) = 0;$ (22)

$\sum_{v\in N} (Z_{vu1} + Z_{vu2}) = 1;$ $\quad \forall u \in M \setminus \{s\}$ (23)

$\sum_{v\in N} (Z_{vu1} + Z_{vu2}) \le 1;$ $\quad \forall u \in N \setminus M$ (24)

$\sum_{v\in N} (Z_{vu1} + Z_{vu2}) \le (n-1) \sum_{v\in N} (Z_{vu1} + Z_{vu2});$ $\quad \forall u \in N \setminus M$ (25)

$\sum_{v\in N} F_{vu} - \sum_{v\in N} F_{uv} = \sum_{v\in N} (Z_{vu1} + Z_{vu2});$ $\quad \forall u \in N \setminus \{s\}$ (26)

$Z_{vu1} + Z_{vu2} \le F_{vu} \le (n-1)(Z_{vu1} + Z_{vu2});$ $\quad \forall u \in N \setminus \{s\}, v \in N$ (27)

Wireless Advantage Property

$p^v_{max} \ge P_v \ge r^{\alpha}_{uv} \cdot Y_{vu} / 2\pi;$ $\quad \forall v, u \in N$ (28)

$0 \le Y_{vu} \le 2\pi (Z_{vu1} + Z_{vu2});$ $\quad \forall v, u \in N$ (29)

$Y_{vu} \le \theta_v;$ $\quad \forall v, u \in N$ (30)

$\theta_v - Y_{vu} + 2\pi (Z_{vu1} + Z_{vu2}) \le 2\pi;$ $\quad \forall v, u \in N$ (31)

Antenna Coverage Property

$A_{vu}\varphi_v - \theta_v + B_{vu} Z_{vu1} \le C_{vu};$ $\quad \forall v, u \in N$ (32)

$2\varphi_v - \theta_v + (4\pi - 2\alpha_{vu}) Z_{vu2} \le 4\pi;$ $\quad \forall v, u \in N$ (33)

$2\varphi_v + \theta_v - 2\alpha_{vu} Z_{vu2} \ge 0;$ $\quad \forall v, u \in N$ (34)

$0 \le \varphi_v < 2\pi;$ $\quad \forall v \in N$ (35)

$\theta_v^{\min} \le \theta_v \le \theta_v^{\max};$ $\quad \forall v \in N$ (36)

Integrality Property

$Z_{vu1} \in \{0, 1\}, Z_{vu2} \in \{0, 1\};$ $\quad \forall v, u \in N$ (37)

Figure 5. MILP model for Problem MEM

6. Performance Evaluation

After the valid problem formulation, in a typical wireless ad-hoc network with no more than 50 nodes, the optimal solution can be always obtained by CPLEX [26], which is a linear, integer and quadratic programming package using simplex method and written in C language. To estimate the time efficiency of our mixed integer linear programming approach, we observe that to solve the MEM Problem based on our MILP model (in Fig. 5) using the CPLEX software package on a MS WIN2000 workstation with a PIII 800-MHz processor and 128 MB memory, the user time is about couple of seconds on each 20-node network example, and less than 10 minutes on each 50-node network example.

We have also evaluated the realistic performance of the heuristic algorithms RB-MIP and D-MIP for many network examples. We specify a typical configuration for the moderate size network with 50 nodes. In each network example, nodes are randomly generated within a square region 1000 meters × 1000 meters. The maximum transmission power can be restricted by the maximum radio propagation range of 300 meters when antenna beamwidth is set its maximal value 2π. One of the nodes is randomly chosen to be the source. Multicast groups of a specified size are chosen randomly from the overall set of nodes. Each antenna can point to any desired direction with an antenna beamwidth subject to $\theta_{\min} \le \theta_v \le 2\pi$. We have only considered propagation loss exponents of $\alpha = 2$.

In all cases, (i.e., for a specified multicast group size m, minimal antenna beamwidth $\theta_{\min}$, and tree algorithm $i \in I$ = {RB-MIP, D-MIP, OPT}, where OPT denotes the branch-and-bound algorithm, which can be obtained in a timely manner based on our MILP model.), our results are based on the performance of 100 randomly generated networks.

6.1. Minimum Tree Power

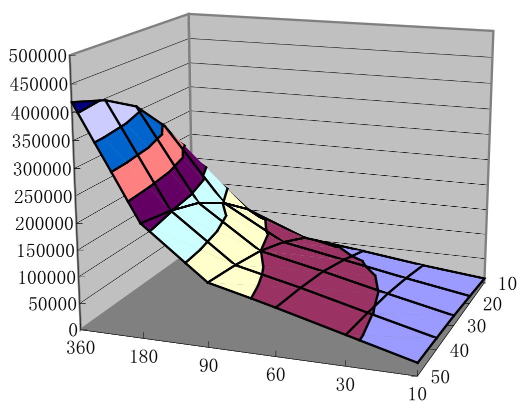


Figure 6. Mean total tree power (z-axis) as a function of multicast group size (y-axis) and minimal antenna beamwidth (x-axis) in 50-node networks.

Let Q_i be the actual total power using algorithm-i. The first set of experiments explores how the *minimum tree power* Q_{OPT} changes with different minimal antenna beamwidth and multicast group size. This performance metric shows the advantage offered by the use of narrow beams compared to the use of omni-directional antennas.

Figure 6 depicts graphically the mean total tree power of the algorithms we have studied over different connected network topologies with 50 nodes. The x-axis represents the minimal antenna beamwidth, the y-axis presents the multicast group size, and the z-axis is the mean of total tree power using different algorithms. The experiment results verify the advantages offered by the use of directional antennas. We also have the following observations:

(1) When the minimal antenna beamwidth is relatively small ($\theta_{\min} < 90°$), the total tree power is slowly increasing with the increment of $\theta_{\min}$ and m.
(2) When the minimal antenna beamwidth is relatively large ($\theta_{\min} > 90°$), the total tree power is increasing much faster with the increment of $\theta_{\min}$ and m.

6.2. Normalized Tree Power

To facilitate the comparison of different algorithms over a wide range of network examples, we use the notion of the *normalized tree power* Q'_i of each network example, defined as the ratio of actual total energy consumption using heuristic algorithm-i to the optimal solutions, i.e. $Q'_i = Q_i/Q_{\text{OPT}}$. This metric provides a measure of how close each algorithm comes to providing the lowest-power tree.

Table 2 summarizes the *normalized tree power* for the four algorithms on networks with 50 nodes, various multicast group sizes, and various minimal antenna beamwidth. We list mean and variance of the normalized tree power as (mean, variance) for each algorithm-i in the table. As noted above, the normalization is taken with respect to OPT. We observe from Table 2 that, for all the cases, D-MIP provides much better performance than RB-MIP both in terms of mean and variance. Figure 7 illustrates graphically the mean normalized tree power as a function of multicast group size and minimal antenna beamwidth on 50-node network examples. We have the following observations based on Table 2 and Fig. 7.

(1) D-MIP performs much better than RB-MIP when $\theta_{\min} < 180°$ for any multicast group size, and both converge to the same performance when $\theta_{\min} > 180°$. This is just as expected since RB-MIP and D-MIP degenerate to MIP when using omni-directional antennas.

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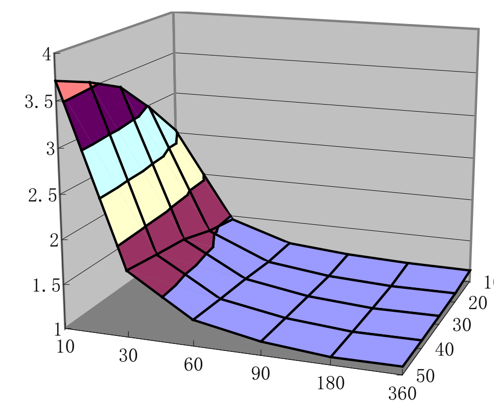
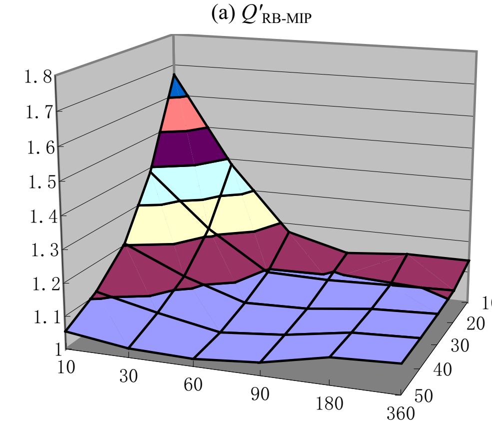
Table 2. Normalized Tree Power in 50-node networks

m	RB-MIP	D-MIP	RB-MIP	D-MIP	RB-MIP	D-MIP
	$\theta_{min} = 10°$		$\theta_{min} = 30°$		$\theta_{min} = 60°$	
10	(2.530, 0.4371)	(1.668, 0.1122)	(1.455, 0.0459)	(1.382, 0.0496)	(1.228, 0.0160)	(1.171, 0.0212)
20	(2.971, 0.2150)	(1.386, 0.0277)	(1.526, 0.0271)	(1.217, 0.0089)	(1.222, 0.0058)	(1.094, 0.0025)
30	(3.358, 0.2315)	(1.204, 0.0083)	(1.641, 0.0342)	(1.120, 0.0028)	(1.254, 0.0081)	(1.058, 0.0010)
40	(3.557, 1.1181)	(1.111, 0.0199)	(1.690, 0.0274)	(1.068, 0.0014)	(1.267, 0.0073)	(1.032, 0.0006)
50	(3.713, 0.1907)	(1.054, 0.0026)	(1.744, 0.0259)	(1.028, 0.0006)	(1.291, 0.0067)	(1.023, 0.0003)
	$\theta_{min} = 90°$		$\theta_{min} = 180°$		$\theta_{min} = 360°$	
10	(1.170, 0.0104)	(1.121, 0.0119)	(1.142, 0.0103)	(1.138, 0.0083)	(1.138, 0.0084)	(1.138, 0.0084)
20	(1.134, 0.0033)	(1.091, 0.0030)	(1.101, 0.0027)	(1.097, 0.0021)	(1.103, 0.0027)	(1.103, 0.0027)
30	(1.138, 0.0033)	(1.063, 0.0009)	(1.090, 0.0012)	(1.086, 0.0010)	(1.095, 0.0013)	(1.095, 0.0013)
40	(1.143, 0.0034)	(1.050, 0.0006)	(1.085, 0.0012)	(1.082, 0.0011)	(1.091, 0.0013)	(1.091, 0.0013)
50	(1.151, 0.0024)	(1.039, 0.0003)	(1.086, 0.0010)	(1.081, 0.0009)	(1.090, 0.0011)	(1.090, 0.0011)



(a) $Q'_{\text{RB-MIP}}$



(b) $Q'_{\text{D-MIP}}$

Figure 7. Mean normalized tree power (z-axis) as a function of multicast group size (y-axis) and minimal antenna beamwidth (x-axis) in 50-node networks.

(2) The mean normalized tree power of RB-MIP is decreasing with the increment of θ_{min} for any multicast group size (in Fig. 7b), and its performance degrades rapidly when θ_{min} becomes smaller than 60°.

(3) The mean normalized tree power of D-MIP is quite sensitive to the multicast group size (in Fig. 7b). Its performance degrades rapidly when the multicast group size decreases especially when $\theta_{min} < 180°$.

(4) OPT would save energy up to more than three times compared to the RB-MIP algorithm in small antenna beamwidth, and more than 60% compared to the D-MIP algorithm in small multicast group size.

7. Conclusion

In this paper we present a constraint formulation for the minimum-energy multicast problem in wireless ad hoc networks with adaptive antennas. Based on the analysis on the properties of minimum energy multicast tree, the problem can be characterized in a form of mixed integer linear programming problem. Many application scenarios can be solved efficiently based on the formulation using branch-and-cut or cutting planes techniques. The optimal solutions can be used to assess the performance of heuristic algorithms for mobile networks by running them at discrete time instances.

A major challenge, and a topic of continued research, is to extend our analytical model to large-scale networks with hundreds of nodes. A near optimal solution can be found in a polynomial time using the Lagrange Relaxation and sub-gradient techniques [27] based on our formulation. Furthermore, it is important to develop the distributed algorithms of MEM to cope with the dynamic topologies.

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