Comparing the structure of power-law graphs and the Internet AS graph

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Abstract— **In this work we devise algorithmic techniques to compare the interconnection structure of the Internet AS Graph with that of graphs produced by topology generators that match the power-law degree distribution of the AS graph. We are guided by the existing notion that nodes in the AS graph can be placed in tiers with the resulting graph having an hierarchical structure. Our techniques are based on identifying graph nodes at each tier, decomposing the graph by removing such nodes and their incident edges, and thus explicitly revealing the interconnection structure of the graph. We define quantitative metrics to analyze and compare the decomposition of synthetic power-law graphs with the Internet-AS graph. Through experiments, we observe qualitative similarities in the decomposition structure of the different families of power-law graphs and explain any quantitative differences based on their generative models. We believe our approach provides insight into the interconnection structure of the AS graph and will find continuing applications in evaluating the representativeness of synthetic topology generators.**

Index Terms—**Internet Topology, AS Graph, power-law graphs**

I. INTRODUCTION

In recent years, a significant amount of work has focused on understanding the properties of the Internet Autonomous System graph. In their pioneering study which compared graph-based models for the Internet topology, Zegura *et al.* [24] identified that the Internet has a *non-random structure* that was not captured by then-existing topology generators. Specifically, they emphasized the presence of an *hierarchy* in the manner in which *Autonomous Systems* [1] were interconnected. Subsequently, another important set of observations was reported by Faloutsos *et al.* [10], regarding the statistical properties of some metrics of the Internet graph. Using measurements of the connectivity of the Internet nodes at both the router and the AS level, they found that several graph metrics such as the distributions of node degrees, the degree ranks of the nodes, and the number of nodes within h hops of each other, could be described by power-laws. The presence of power-laws in the Internet graph is now considered to be empirically well established. Similarly the notion of an hierarchical structure in the Internet graph, and the presence of a routing hierarchy has also been commonly noted in literature [24], [13]. These observations have been a starting point for a flurry of work on developing synthetic Internet topology generators. Generators such as *Tiers* and *Transit-Stub* create graphs with an explicit hierarchical structure as a model for the Internet. From another starting point, work by Barabasi *et al.* [5], Aiello *et al.* [2] and others [7], [17], [15] has led to topology generators that aim primarily to match the power-law degree distribution of the Internet graph.

There have been several studies comparing power-law degree based generators and the Internet graph. In [18], the authors compared the graphs produced by these generators based on metrics such as power-law exponents, degree rank, hop-plot and eigenvalue distributions. In [7] the authors introduced the clustering coefficient and the median shortest path length as useful metrics for distinguishing among the different topology generators. And, most recently, Tangmunarunkit *et al.* [22], carried out an extensive comparison study, using a wide range of metrics, including expansion (neighborhood size), resilience (size of a cut-set for a balanced bipartition) and distortion (minimum-communication-cost spanning tree). Although there isn't yet a consensus on which of the above-mentioned metrics are the most important, one common property of these metrics is that they do not give insight into the structure of the graph. Our work in this paper addresses the following question: how well do power-law graphs capture the *interconnection structure* (such as hierarchy) of the Internet graph?

One possible way of thinking about the "structure" of a graph is by comparison with canonical topologies such as a star, a mesh or a binary tree. In our context, there is a widespread belief that the Internet graph is hierarchical in structure. In this study, we use algorithmic techniques to explore the structural properties of power-law graphs with respect to the Internet graph, guided by existing notions of how ASes connect to each other. Using the notion that ASes are arranged in tiers, we identify nodes at each tier, and recursively decompose the graph to expose its interconnection structure. We define metrics of the resulting decomposition, which then allow us to quantitatively and statistically compare the structural properties of power-law graphs and the Internet graph. The properties of the decompositions of these graphs allow us to also examine questions like whether the graph is hierarchical in nature. We observe, and through statistical tests exhibit, similarities in the decomposition structure of power-law graphs as compared to the Internet AS graph. We also find that both the skewed-degree distribution and the degree of preferential connectivity play a role in defining the decomposition structure of these graphs. These

[1] An AS is a group of routers and end-hosts that have common routing policies, with respect to the rest of the Internet.

two properties cause a large number of nodes (which also have small-degrees) to directly depend on the high-degree nodes for connectivity to the rest of the graph, and also contribute to the resiliency of the graph by causing high-degree nodes to connect with each other.

The rest of this paper is organized as follows. In Section II, we elaborate on existing notions of the structure of the Internet graph. Section III introduces algorithmic techniques which leverage knowledge of structural properties of the Internet graph and apply them to power-law graphs and the Internet graph, to achieve an hierarchical decomposition of these graphs. We describe some metrics of interest which can be used to compare the decompositions of the different graphs. We also describe some related work which has also examined issues like the presence of an hierarchy in the AS graph, and in power-law random graphs. In Section IV we introduce the graph families that we examine as part of this study. Section V discusses the results from the decomposition, and their implications for the structure of the graphs. We also examine the decompositions of the different families of graphs, and compare the similarities in more detail through statistical tests. We describe future directions for this work, and conclude in Section VI.

II. Structure of the AS graph

The Internet is composed of a collection of administrative domains called *Autonomous Systems*. Based on the properties of the routes starting, ending or passing through an AS, it can be classified as either a *stub* or a *transit* AS. As defined in [24], a stub AS is one such that the path connecting any two end-hosts, *u* and *v*, in the Internet traverses this AS only if either *u* or *v* belongs to this AS. Transit ASes do not have this restriction, and can hence serve as an intermediary in any path. Stub ASes usually correspond to universities or large commercial organizations, which rely on transit ASes for connectivity to the rest of the Internet. They themselves do not offer such a service to any other AS. Transit ASes, in Internet terms, are *service providers* and are typically regional and national level ISPs, or *backbone* networks. They offer connectivity to several stub ASes and are also well-connected to each other.

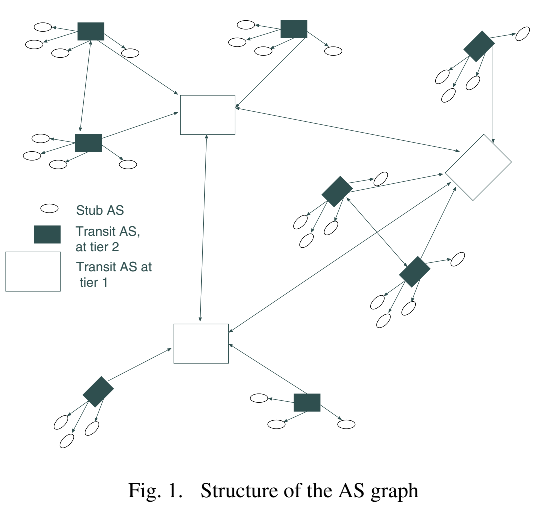

Fig. 1. Structure of the AS graph

The notions of transit and stub domains suggest a structure as to how ASes connect in the Internet. Stub nodes connect with one or more transit nodes, and all paths originating or culminating in a Stub node must traverse these *provider* transit nodes. Also, transit domains can be either providers to, or *customers* of other transit nodes. Thus, based on these provider-customer relationships, each AS in the Internet can be considered as belonging to a particular tier [2], with the ASes at the highest tier being the transit domains that have no providers (the so called Tier-1 providers). Stub ASes are completely dependent on the transit nodes in the tier above for connectivity to the rest of the Internet (to a lesser extent, this is true also of lower tier transit ASes. However transit ASes can also route some traffic through *peering* relationships they have with other transit ASes in the same tier.).

The arrangement of nodes into different tiers, as described above, and the relationship between the transit and stub ASes also provides a possible hierarchical structure of the Internet graph. One can conceive of the Internet as composed of a set of transit ASes at the top of the hierarchy, offering connectivity to both transit and stub ASes, at the next tier. The second tier transit ASes themselves provide connectivity to other ASes below them, and so on. In Section III, we introduce techniques that allow us to examine the structure of the graph based on this notion.

III. Exploring structure

We now describe the technique we use to understand and compare the structure of the Internet AS graph with those of graphs that have been generated to simply match the power-law degree distribution of the AS graph. Our efforts are guided by the interconnection properties of the transit and stub nodes in the Internet AS graph, as described in the previous section.

We begin by describing a criterion to identify the root-level transit nodes. We then decompose the graph by removing these nodes and their incident edges from the graph. The procedure is then repeated recursively, over every connected component of the resulting graph. A concise description of the procedure is as follows:

1) Given an input graph, G, select a set of nodes to be removed.
2) Compute the connected components (CCs) of the graph G obtained after removing the selected nodes.
3) Repeat the procedure recursively on the resulting CCs, until the number of nodes that can be removed is less than 1.

This technique serves two objectives. It assigns the nodes of the graph to a particular level (or tier): at each level of decomposition, the nodes selected for removal belong to that level. Also, the decomposition of the graph at each level exposes the interconnection structure among nodes within that level.

A key aspect of this technique is the criterion used to select nodes to be removed at each level of the decomposition. Our first choice for this metric is based on *node degree*. We order the nodes in each connected component in descending order of degree and choose a fixed fraction α of the highest degree nodes to remove. We justify using this metric by looking into the degree of the ASes in the Internet in relation to the tier to

[2] or "level"; we use the terms "level" and "tier" (to which a node belongs) interchangeably in this paper.

	Average degree
Tier 1	614.29
Tier 2	19.30
Tier 3	6.93
Tier 4	4.30

TABLE I
AVERAGE DEGREE OF ASES IN DIFFERENT TIERS

which they belong. Using the inference techniques developed in [12], we first arrange the ASes into their respective tiers. The average degrees of the *Provider* ASes[3] appears in Table I. As can be observed, the average degree of the ASes decreases as we go down the tiers, suggesting a positive correlation between a node's tier and its degree. In Figure 2 we plot the ASes, arranged in the x-axis as per the tier to which they belong, with their respective degrees in the y-axis. Looking at this figure we observe that in the actual Internet, an AS with a smaller degree can be placed at a tier higher than an AS with a higher degree. This indicates that node degree does not completely capture the semantics of how ASes are placed in the real AS graph.

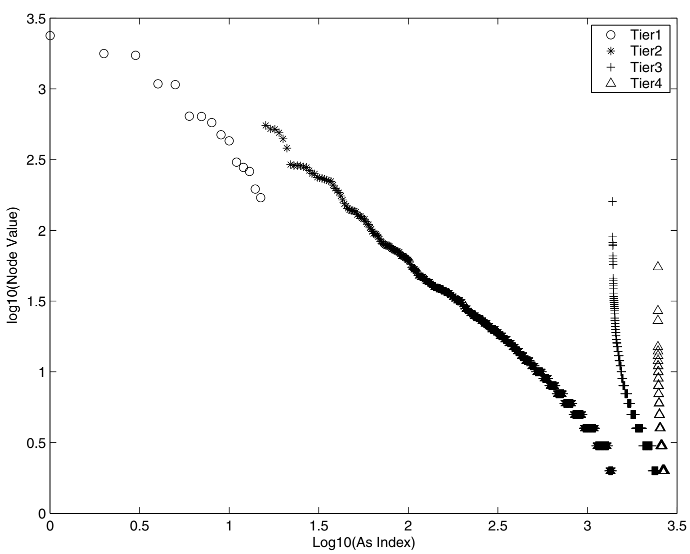

Fig. 2. Degree of ASes in different tiers

We considered two other metrics. The first is *node rank*, a notion adapted from *PageRank*, a metric developed in [19] to rank web pages. (A similar notion to capture the "authority" of web pages was also developed in [16].) Let the nodes of the graph be denoted $1, 2, ..., N$. Let $d(j)$ denote the degree of node j, and let $Nb(j)$ denote the set of j's neighbors in the graph. Then $r(j)$ is the steady-state probability of visiting node j while performing a random walk in the graph where each outgoing link from a node is equally likely to be chosen, $r(j)$ satisfies

$$r(j) = \sum_{i \in Nb(j)} \frac{r(i)}{d(i)}$$

[3]The provider ASes correspond to the nodes we seek to remove at each level of decomposition.

$$\sum_{j=1}^{N} r(j) = 1$$

This metric is computed iteratively. Initially all nodes are assigned the rank-value $1/N$. At each step, the ranks of the nodes are computed based on the above definition and normalized such that ranks of all nodes sum to 1. This procedure is repeated until the values of the node ranks converge[4]. Our third choice of the metric is *node stress*, a metric that is based on the *link value* metric defined in [22]. The *stress* of a node v in a graph, is a measure of the number of node-pair shortest paths that pass through v. We define the *node stress* of any node in the graph to be the size of its traversal set, normalized by the number of all node-pairs in the graph. The traversal set of a node v is the set of all node-pairs (s, d) in the graph, whose shortest path traverses node v. After computing the *Node Stress* and *Node Rank* of all nodes in the input graphs, we found that there is a very strong correlation between the degree of the node and its *stress* or *rank*. Therefore, we do not obtain any new information about the interconnection semantics of the input graphs using these metrics. Hence we restrict ourselves to presenting results of the decomposition procedure, using only the node degree metric.

We describe some metrics that allow us to characterize a decomposition quantitatively.

- N_{CC} quantifies the number of CCs at any level of the decomposition.
- σ_{CC} is the standard deviation of the sizes of the CCs at each level.
- D the depth of the decomposition, i.e., number of levels of recursion until we do not find any more nodes to remove from the CCs at that level of recursion.
- We also examine the distribution of sizes of the CCs at each level of decomposition, and the distribution of node degrees in the largest CC at each level of decomposition.

In addition to exposing the interconnection structure of the input graph, this decomposition procedure allows us to examine whether the graph has hierarchical properties. Although there is no precise general notion of hierarchy, one way of thinking about it is in terms of classical hierarchical graphs, such as rooted trees. A key characteristic of such graphs is that a path from a node *v* in this graph, to any other node (which is not a descendant of *v*) passes through the parent of *v*. Hence the removal of nodes belonging to a higher tier would partition the graph of the remaining nodes at tiers below. One can observe an appropriate parallel of this characteristic in the Internet graph. The transit nodes with no providers would form the "root" tier, and other transit and stub nodes would successively be arranged in the tiers below, based on the relationships among these ASes. However, it is not clear if this structure is hierarchical in the sense of the rooted trees, described above. It is worth noting that breaking a graph into tiers does not necessarily shed any light into whether it has an hierarchical structure, since tiering does not impose the notion that nodes in a tier depend on the

[4]Another way of computing the Node Rank is to consider the stochastic matrix derived from normalizing the columns of the adjacency matrix of the graph under consideration. The Node Ranks of the graph nodes are then the values in the principle eigenvector of the stochastic matrix [19].

one above them for connectivity. Nodes within a tier could be well connected within themselves, and perhaps also to nodes in tiers above the one immediately on top of them. In fact, a tiering could be induced in any graph, hierarchical or not. We soon discuss this in more detail, and explore to what extent the graphs we examine exhibit such hierarchical characteristics.

A. Related studies

There has been some previous work in *characterizing* the hierarchy of the Internet graph. In [13], the authors observed that ASes can be divided into four classes, with significant variation of average degree between the classes. The authors propose a tiering induced upon the AS graph, with the class of ASes with the highest average degree at the top tier, followed by others in descending order of the average degree. A similar tiering of ASes has also been introduced in [12], [21], but using the logical relationships between the ASes. In [12], the authors create a logical tiering based on the Customer-Provider relationship between ASes, with a Provider AS assigned a tier higher than its Customer. A similar approach has been suggested by [21], using logical relationships and some notion of node interconnectivity to distinguish adjacent tiers. A recent work [22] also considers the hierarchical characteristic of the Internet graph, and the degree to which the degree based generators capture this property. Their metric of choice is the distribution of *link-values*, where the value of a link can be roughly defined as the number of node pairs whose shortest paths traverse this link. The authors study the distribution of link-values for the Internet router and AS level graphs, power-law graphs and some canonical graphs. Examining the distribution of this metric for different topologies and comparing it with the distribution of the classical tree topology, the authors qualitatively compare the relative degrees of hierarchy of graphs of different topologies. Although this metric serves a useful purpose in distinguishing among the different graphs, it falls short of answering several questions. The distribution of link-values does not lend any insight into the interconnection structure of the graph which would have led to this distribution. Also, observing the distribution of link-values does not answer some questions about the nature of the hierarchy, whether it is balanced, how deep is it etc. In other words, we are not able to "visualize" the structural properties of these graphs.

IV. Input graphs

We now introduce the set of graphs that we examine. The Autonomous Systems topology has been created from the BGP routing tables collected at the *route-views* server (`route-views.oregon-ix.net`). The *route-views* data set consists of routing tables exported from BGP routers of various Tier-1 ASes, and provides one of the most comprehensive views of the current Internet. Since BGP is a path vector protocol, the routes advertised in these tables can be used to infer AS adjacencies and thus the AS graph. One problem with this approach is that ASes selectively announce routes to other ASes based on the contractual agreements between them. Hence, if we have information only from some select BGP routers, we may miss out on some advertised routes (and hence AS adjacencies) in the AS graph. In a recent work [8] the authors have discussed this problem, and have extended the *route-views* data set with BGP routing tables of a few other ISPs, and entries from the Internet Routing Registry. They found the AS topology constructed from this data set to have a significant number of extra edges. Moreover, the degree distribution of this augmented AS graph was found to not conform to a strict power-law distribution. The authors also examined the nature of these missing edges between the the two data sets, and found that most were either peer-peer edges between lower-tier ASes, or customer-provider edges. We applied our decomposition techniques on AS graphs constructed from both the *route-views* and the extended data set. We found (as will be discussed in detail in Appendix A) that our results and observations hold across these two different topologies. Moreover, a major source of the extra edges in the extended topology (nearly 72% [8]) are customer-provider links. These are from a multi-homed AS to its providers, and may be fail-over links that are used only when the primary link is not operational. This brings into question the relevance of some of the missing edges between the two data sets. Based on these points, we chose to adopt the *route-views* data set as our AS topology of reference for this study. Finally, even though the degree distribution of the AS graph constructed from the *route-views* data set may not follow a strict power-law, these graphs share important characteristics. Both have a highly skewed degree distribution, and common generative principles such as a notion of preferential connectivity. These characteristics, as we will soon observe, play a key role in determining the decomposition structure of these graphs. Thus, we argue, power-law graphs are relevant models for comparison with the AS graph.

We represent the AS level topology of the Internet by a graph, $G =< V, E >$, where each $v \in V$ denotes an AS and each $e \in E$ is an undirected inter-AS connection inferred from the routing table data. We use the routing tables from May, 2004 to construct the AS topology. Next, we consider two variants of power-law based degree generators,

- *PLRG* (power law random graph) is a generator developed in [2]. Given a target number of nodes N and a power-law exponent β, PLRG first assigns degrees to all the nodes drawn from this power-law distribution. It then randomly matches degrees among all the nodes. This procedure may produce graphs which are not connected, as well as graphs that have have self-loops and duplicate links. It has been shown in [2] that there exists a giant connected component, for a large range of values of β. We hence search for this giant connected component and remove all duplicate links and self-loops.
- *GLP* (generalized linear preference) [7] extends the technique proposed in [5]. Starting with a small set of core nodes, the technique incrementally constructs the graph. At each step, one of two operations is probabilistically chosen *(i)* adding a new node along with m links, or *(ii)* adding m new links without a node. In both cases, the links are connected to existing nodes with a probability that is proportional to their degrees.

As a means of comparison with classical random graphs, we also choose topologies generated by the *Waxman* generator [23]. The classical Erdos-Renyi random graph model [6] assigns a uniform probability for creating a link between any

	AS Graph	PLRG	GLP	Waxman
Number of Nodes	17611	17525	17611	17611
Number of Edges	38015	43377	27199	35222

TABLE II
CHARACTERISTICS OF THE INPUT GRAPHS

	AS Graph	PLRG	GLP	Waxman
Depth	9	14	6	37

TABLE III
DEPTH OF THE DECOMPOSITION

pair of nodes. The Waxman generator extends the classical model by randomly assigning nodes to locations on a plane and making the link creation probability a function of the Euclidean distance between the nodes.

Table VII describes some specific properties of the studied graphs. We use the Autonomous System graph inferred from the Route-view server's routing table data collected on May, 2004, as our reference graph. The topologies generated using other generative mechanisms aim to match the AS graph in terms of numbers of nodes and edges. We generated 100 instances of these graphs with a different initial seed for each instance; the table presents the average quantities computed over all these instances. The empirical complementary cumulative distribution of the node degrees of the AS graph follows a power-law, with an exponent $\beta = -1.125$. The graphs generated by the PLRG and GLP generators closely (although, not exactly) match this exponent.

V. RESULTS FROM THE DECOMPOSITION

Initially we choose a fixed value of $\alpha = 0.01$ [5] the fraction of nodes to be removed from the CCs of the graph at each level of decomposition. We later use different values of α, at different levels of decomposition and briefly discuss the differences in results in Appendix C.

Let us first consider the decomposition of the graphs with power-law degree distributions, namely the AS graph, PLRG and GLP graphs. We start by removing the top 1% highest degree nodes and their edges from the input graph. We then identify the CCs in these decomposed graphs, and repeat the procedure recursively. In all three classes of graphs, we observe non-trivial-sized CCs until a level of recursion ranging from 6 - 14 as shown in Table III.

[5] Studies have reported that about 1% of all ASes in the Internet have no providers. About 20 of these ASes have been found to form (almost) a clique, which would be another (lower) estimate of the Tier-1 ASes.

	AS Graph	PLRG	GLP	Waxman
Level 1	8267	7651	11414	7
Level 2	1190	697	380	18
Level 3	749	492	238	23
Level 4	375	392	145	32
Level 5	273	326	73	39

TABLE IV
NUMBER OF CCS AT THE FIRST FIVE LEVELS OF THE DECOMPOSITION

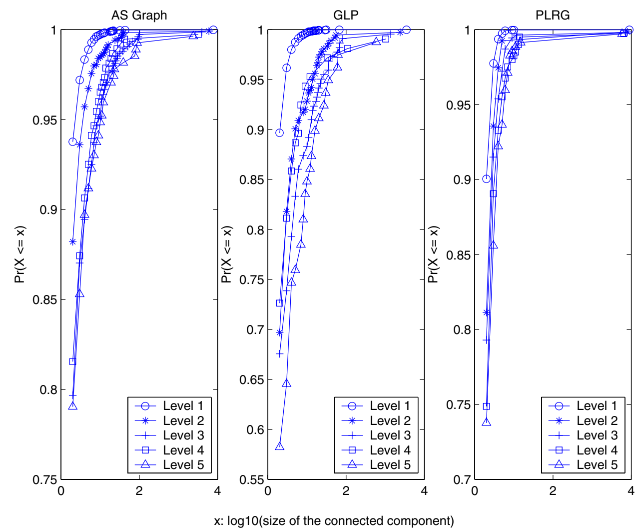

Fig. 3. CDF of the sizes of CCs of first 5 levels of decomposition

	AS Graph	PLRG	GLP	Waxman
Level 1	45%	50%	21%	99%
Level 2	76%	88%	73%	99%
Level 3	66%	90%	66%	99%
Level 4	80%	90%	54%	99%
Level 5	75%	90%	51%	99%

TABLE V
PERCENTAGE OF NODES IN THE LARGEST CC IN THE FIRST 5 LEVELS OF DECOMPOSITION

One observes in Table IV that the removal of the selected nodes decomposes the graph into a large number of CCs. The distribution of the sizes of these CCs is, however, highly skewed, with a large fraction of CCs being trivial. This can also be observed in Figure 3, which plots the CDF of this distribution, and indicates that between 80-90% of all CCs have either 1 or 2 nodes. Although we have not yet carried out statistical tests to back this claim, visually these distributions seem similar, for the AS, PLRG and GLP graphs. The disparity in the sizes of the CCs is also reflected in Table VI which computes σ_{CC}, which are extremely high across the three families of graphs. The next common characteristic of the decomposition is the existence of a "giant" CC at each level of decomposition. This giant CC, as shown in Table V, contains 21-50% of nodes in the first level, and 50-90% of nodes in subsequent levels. However, there is also a substantial difference in the relative sizes of the giant CC in the first level of decompositions of GLP graphs (21%), AS Graphs (45%) and PLRG Graphs (50%). We consider the reasons behind this difference imminently.

The decomposition structure of the power-law graphs is strikingly different from that of the Waxman graph. Removing the highest-degree nodes from that graph fails to decompose the graph in any significant measure. As can be seen from Table IV, a very small number of CCs are formed at each level of the de-

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	AS Graph	PLRG	GLP	Waxman
Level 1	86.62	104.25	32.63	8715.5
Level 2	174.57	309.71	126.54	4310.5
Level 3	145.85	308.50	92.82	4135.7
Level 4	165.62	322.41	100.34	2807.9
Level 5	144.90	312.61	66.58	2566.2

TABLE VI
STANDARD DEVIATION OF THE SIZE OF CCS IN THE FIRST 5 LEVELS OF THE DECOMPOSITION

composition, with one CC comprising almost all nodes of the graph, and a few trivially sized CCs. This is not surprising since in a classical Erdos-Renyi random graph, most nodes have a degree close to the mean degree of the graph, with the maximum degree of the graph being orders of magnitude smaller than the maximum degrees in a similar sized power-law graph. Hence, the removal of the highest degree nodes has a very limited impact on the structure of the graph. This difference also translates into a significantly larger depth of decomposition for the Waxman graph.

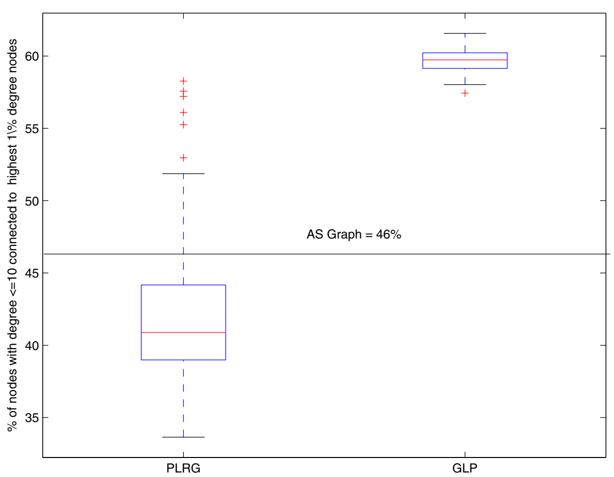

Fig. 4. A measure of the preferential connectivity in PLRG, GLP and AS Graphs

We now explain why there is a significant difference in the relative sizes of the giant CCs, in the first level of decomposition among the three graph families. We know that these graphs have a very skewed degree distribution (80% - 90% of nodes in the three graph families have degree ≤ 3, and 95% of node have degrees ≤ 10) and the node-degrees of the highest-degrees nodes are orders-of-magnitude higher than the average node degree. We observe also that most nodes that are not in the giant CC belong to trivial sized connected CCs. One explanation for this could be that nodes with small degrees comprise most of the neighbors of nodes with the largest degrees. Removing the highest-degree nodes could disconnect many of these low-degree nodes, which would then form trivial-sized connected CCs. In order to understand how this observation helps explain the difference in the relative sizes of the giant CCs, we examine Figure 4 which is a box-plot of the percentage of nodes with degree ≤ 10, that are connected only to the removed highest-degree nodes. We notice that GLP graphs have, in general, a much higher fraction of small-degree nodes connected solely to the highest degree nodes (median: 60%), as compared to the AS Graph (median: 46%), which in turn has a higher percentage of such nodes than PLRG graphs (median: 41%). Removal of the highest degree nodes in the GLP graphs thus disconnects a greater percentage of nodes in the remaining graph, as compared to PLRG and AS Graph. This directly leads to a smaller size of the giant CC in the GLP graphs. On the other hand, PLRG graphs, which have the smallest percentage of small-degree nodes connected to the highest-degree nodes, have larger giant CCs resulting from the decomposition. This difference in structure can be explained by the differences in the connectivity models of these graphs. PLRG graphs are based on a linear-preference connectivity model, while it has been reported in [9] that in the Internet, new ASes have a much stronger preference to connect to large-degree ASes than predicted by the linear preference model. GLP graphs have been designed explicitly to incorporate this greater than linear preference for new ASes, in order to connect with ASes with large degrees.

To further support our observation that connection preferences of lower-degree nodes determine the sizes of the giant CCs, we have also constructed power-law graphs with a preference for high-degree nodes, to connect with the lowest-degree nodes (as described in Appendix B). Since nearly all the smallest-degree nodes in such graphs are connected to the highest-degree nodes, the decomposition results in no giant CCs at any level of decomposition.[6]

We now address the question of whether these graphs are hierarchical. First, the graphs' decompositions tend to be highly imbalanced, with the size of the largest CC being orders of magnitude larger than the average size of the CCs. Second, there are many CCs at each level of the decomposition. Referring to our previous discussion, a criterion we stipulated for a graph to be hierarchical was that nodes (aside from those in the topmost level) would depend on the nodes in the level above for paths to the rest of the graph. This is true for the decomposition of the graphs we have studied, since members of the (numerous) small CCs are disconnected from the graph upon removal of the nodes at a higher level. Also, these nodes form a substantial fraction of nodes (50% to 79% at decomposition level 1). Thus, based on this criterion, the resulting decomposition does have hierarchical properties. However, the fact that there exist numerous trivial-sized CCs is a result of the skewed degree distributions of these graphs and their preference for small-degree nodes to connect with large-degree nodes. Moreover a significant fraction of nodes at each decomposition level remain part of a giant CC, with the relative size of this CC rising to as much as 90% of all nodes at lower levels. Since the presence of this giant CC implies that a large percentage of nodes remain connected even upon the removal of the nodes at the level above, it serves as a counterpoint to our earlier evidence that power-law

[6] This also helps illustrate why a substantial fraction of nodes in the graph remain in one giant CC. In the PLRG, GLP and AS Graphs, because of a preference for nodes to connect with nodes of a higher degree, high degree nodes themselves tend to connect with each other. In the power-law graph deterministically constructed with high degree nodes connecting with the lowest degree nodes (described in Appendix B), there exist few edges between the high degree nodes, resulting in a very low resiliency of these graphs.

graphs have hierarchical properties.

To summarize, the decomposition structures of the AS Graph, and GLP and PLRG power-law graphs seem to be qualitatively similar. All three graphs show a non trivial depth of decomposition, a large number of trivially sized CCs at each level, and a "giant" CC that comprises a significant percentage of the nodes. We have explained the quantitative difference in the size of the CCs as resulting from the differences in their connectivity models. In the next subsection, we focus on the similarities in the decomposition behaviors of these families through statistical tests.

A. Statistical tests comparing the decomposition of PLRG, GLP and AS Graphs

We first consider the distribution of node degrees in the giant CC at each level of decomposition. As can be observed from Figure 5, the initial input graphs from the three sources show a power-law distribution of degrees. However the distributions of degrees in the largest CC of subsequent levels no longer follow a power-law. In fact the tail seems to drop exponentially in the case of all three families. As a first step toward ascertaining if the degrees of the nodes in the giant CC come from the same distribution for the three graph families, we perform a visual statistical test. We plot the quantile-quantile plots (qq-plots) of the degree distributions of nodes in the giant CC from the first level of decomposition. A linear trend in the qq-plot for all the three graphs, considered pairwise, indicates that degree distributions come from at least the "same type" of distribution, albeit with different parameters.

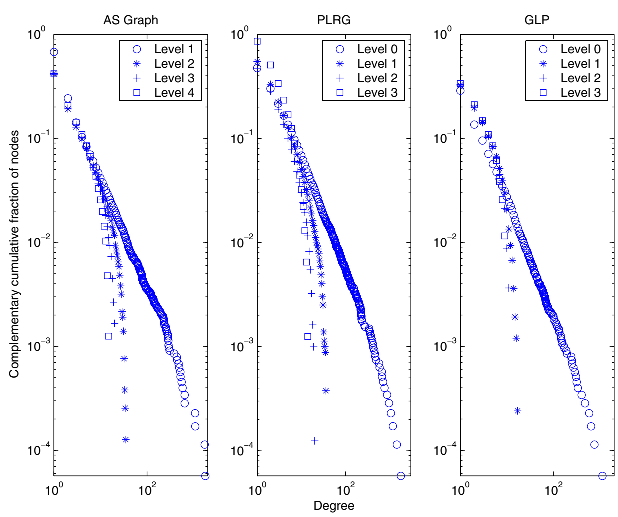

Fig. 5. CCDF of node degrees in the original graph (Level 0), and nodes in the giant CC of the first 3 levels of decomposition (Levels 1 - 3)

Given this visual evidence, we tried to identify candidate analytical distributions that would describe our data. We observe two distinct regions in the log-log plot of this data, a linear body, and an exponentially dropping tail. Hence, we postulate that instead of a single distribution, a hybrid of two different distributions would better describe the entire dataset. As candidate distributions, we choose the Pareto for the body, and the Exponential distribution for the tail. We observe an encouraging visual fit of these distributions in their respective regions in Figure 7. The next step is to test the match through a statistical goodness-of-fit test, for which purpose we employ the Kolmogorov-Smirinov two-sample test. The null hypothesis for this test is that two given input sample-sets are from the same underlying distribution. The test statistic is defined as follows. Let X_1 and X_2 denote the input sample-sets, and let F_1, F_2 be their respective Empirical Distribution functions. The test statistic T is computed as:

$$T = max_{x \in X_1 \cup X_2} |F_1(x) - F_2(x)|$$

We initially found that the null hypothesis was rejected at the 95% confidence level, for some graph instances. Since this could be an artifact of the large number of samples we have available for the test, we redid the KS test with a randomly chosen subset from the entire data set. We now find that the goodness of fit tests succeed for both the Pareto and the Exponential distribution at a 95% confidence level. Based on these tests, we believe that the distributions of node degrees in the largest CCs come from the same family of distributions, for the PLRG, GLP and AS graphs[7].

We now consider if it is possible to compare the parameters of these distributions. We first consider only the parameters of the Pareto distribution, which is fitted to the body of the sample population. For each sample graph from the GLP or the Pareto family, we first need to infer the parameter of the Pareto distribution. To do this, we employ the *least squares* estimator method, as described in [20]. In Figure 8, we box-plot the inferred values of the parameter for the Pareto distribution fitted to the degree of the nodes in the giant CC in the first level of decomposition. For the AS-graph, the value for this parameter is 1.40. From the box-plot we observe that though the inferred parameters for the GLP and PLRG graphs are spread over a range of values, they are fairly close to the inferred value for the AS Graph. It is difficult to make a more precise quantitative comparison between the inferred values of these parameters, given that there is a small range of parameters for which the goodness-of-fit tests do not reject the null hypothesis.

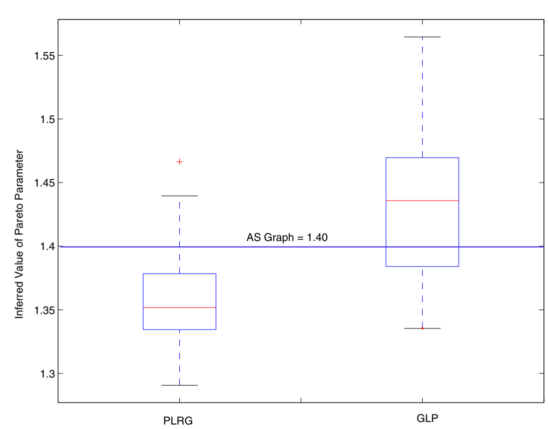

Fig. 8. Box-plot of inferred parameters for the Pareto distribution over GLP and PLRG graphs

[7]We include the qqplots and the plots of fitted distribution of the degree of nodes in the giant CC, from the first three levels of decomposition in the extended version of this paper [14].

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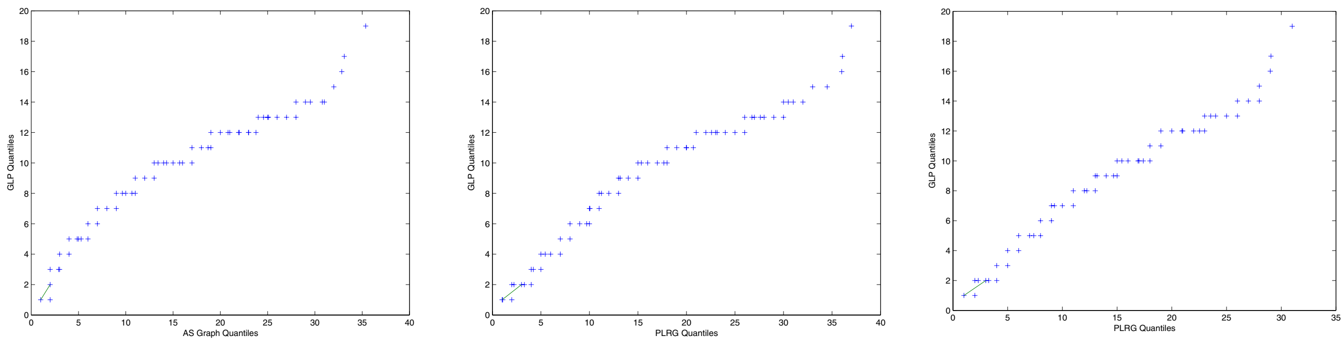

Fig. 6. qq plot of degree of nodes in the giant CC, first level of decomposition

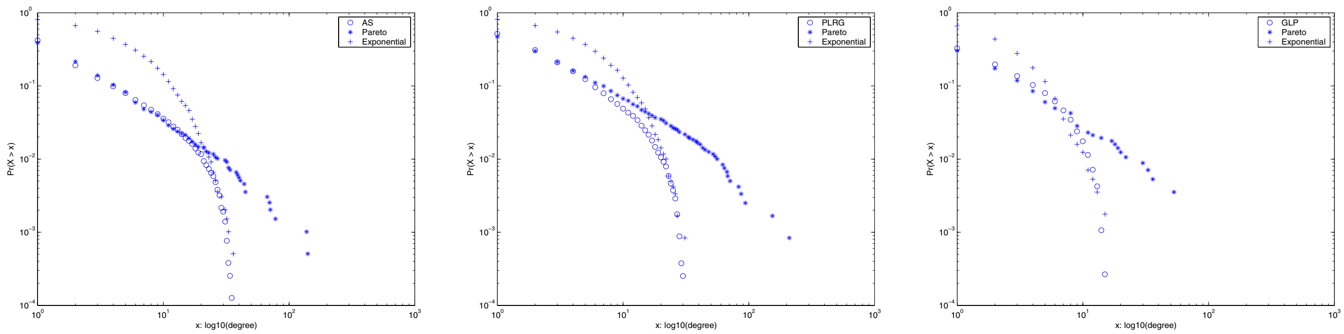

Fig. 7. Fitting Pareto and Exponential distributions to body and tail of node degrees in giant CCs, first level of decomposition

To summarize, through the use of both visual and quantitative goodness-of-fit tests, we observe the metrics used to quantify the decomposition of a graph, are defined by same family of distributions for the AS, PLRG and GLP graphs, indicating that these graphs have a similar and closely related decomposition structure.

VI. Conclusions and Future work

This work was motivated by the notion that, as defined by their business relationships, there exists a structure in the manner in which the Autonomous Systems in the Internet connect with each other. More specifically, the notion that ASes are arranged in tiers, and a particular AS node connects with others, usually through a provider AS in a preceding tier. In this work, we use simple metrics, to identify ASes in a particular tier, and using decomposition techniques, examine the inter-connection structure between them. We then apply these techniques to graphs generated from the PLRG and GLP family of generators. We observe, and validate through statistical tests, that the AS graph, and graphs from the PLRG and GLP families have similar decomposition structure. We also discuss the hierarchical properties, of these decomposition structures.

We observed in Section III, that if we arrange the nodes of the AS graph into different tiers, based upon inferring logical relationships between them, then a node of a lower degree (or lower *node stress*) can be present in tier higher than a node with a larger degree (or *node stress*). Hence these metrics may not capture all the necessary semantics of how nodes in the AS graph interconnect. An interesting issue to look into carefully is how to map the semantics of the logical relationships between ASes on to links in undirected graphs. With respect to our decomposition techniques, this would help in choosing which and how many nodes to extract at each level of decomposition.

Our decomposition techniques can be considered complementary to other metrics defined in previous works [7], [18], [22] to compare Internet topology generators and the AS graph. The approach in this work is new in the sense that it is the first to look explicitly into the inter-connection structure of power-law graphs. Applications of these techniques will include examining graphs generated by new schemes aimed at emulating the AS Graphs, such as [11], [3], [4].

Acknowledgments

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	Route-views	Extended
Number of Nodes	10670	10900
Number of Edges	22002	31180

TABLE VII

CHARACTERISTICS OF THE INPUT GRAPHS

	Route-views	Extended
Level 1	41%	57%
Level 2	76%	82%
Level 3	79%	86%
Level 4	73%	88%
Level 5	76%	90%

TABLE VIII

PERCENTAGE OF NODES IN THE GIANT COMPONENT IN THE FIRST 5 LEVELS OF THE DECOMPOSITION

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APPENDIX

A. Comparing the Route-views and Extended AS topologies

We now compare the results of our decomposition techniques on two versions of the AS graph topology. The first (that we shall term *Route-views*) is derived solely from the BGP routing tables of the *route-views* repository. The second topology (termed *Extended*) is constructed from connectivity information attained from data sources in addition to the `route-views` repository. As we can observe in Table VII, the *Extended* topology has 40% more edges and about 2% more nodes than the *Route-views* topology.

As discussed earlier, the *route-views* data set is based on BGP tables collected from mostly Tier-1 ASes, which may not have information about some links between lower-tier ASes. In order to get around this problem, the extended data-set augments the *route-views* data set with information from BGP routers of several other lower-tier ASes, and using information from the Internet Routing Registry. This additional information, especially that from the routing registry provides us with more inter-AS edges than obtained from the *route-views* data set alone. We actually downloaded instances of the AS topology (from [1]) created using the augmented data set as part of the study done by [8] dating from March, 2001.

Let us now apply our decomposition techniques to both these graphs. We observe that the main difference in the decomposition is that, across all levels of decomposition, the extended topology has a larger relative size of the giant component as compared to the route-views topology. For example, in the first level of decomposition 57% of all nodes are in the giant component in the *Extended* topology as compared to 41% in the *Route-views* topology. Let us examine why this is the case. Firstly, as pointed earlier, several extra edges in the *Extended* topology are links between a multi-homed AS and its providers. It is possible that an AS may spread such links across providers in different tiers. Thus, as an example, upon removal of the highest 1% degree nodes, a customer AS may lose some of its provider links, but could continue to remain connected to the giant component by virtue of also having links with lower-degree (or lower-tier) ASes. In order to verify this notion, we looked into what percentage of nodes with degree ≤ 10 are connected solely to the highest 1% degree nodes. In the case of the *Route-views* topology, nearly 46% of such nodes were connected only the highest degree nodes, and this falls to 39% of nodes in the *Extended* topology. Also, another source of the extra edges are peer-peer links between lower-tier ASes. The existence of such links further increases the resiliency of the giant component across all levels.

Finally, we examined the degree distribution of nodes in the giant components at different levels of decomposition. We found that, similar to the *Route-views* topology, the degree distribution in the *Extended* graph follows the Pareto in the body and the Exponential distribution for the tail, as is shown in Figure 9. We have statistically verified this hypothesis by doing goodness-of-fit tests for these distributions.

To summarize, after applying our decomposition techniques on the augmented data set we observe the decomposition of the *Extended* topology remains qualitatively and statistically similar to the *Route-views* topology despite some quantitative difference in the size of giant connected components at different levels of decomposition.

B. Decomposition over different PLRG constructions

A question we consider is if the similarity of the decomposition in the three family of graphs is simply a byproduct of

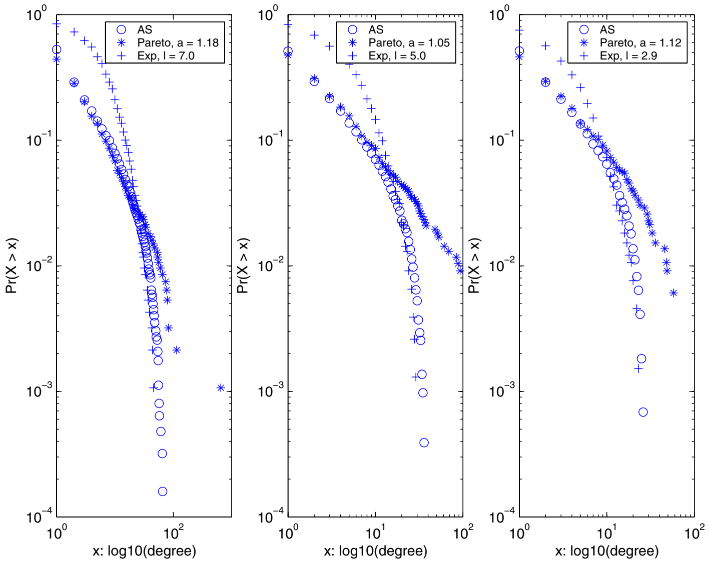

Fig. 9. Fitting Pareto and Exponential distributions to body and tail of node degrees in the giant CC at first 3 levels of decomposition in the *Extended* topology

the fact that they have the same degree distribution? In order to answer this question, we construct three different types of PLRG graphs, each starting with the same node degree distribution, but with nodes interconnected differently. The first type of graph is the standard PLRG graph in which nodes of a certain degree are randomly matched with each other. In the second type of graph (termed PLRG-ascending), we connect nodes in the following way. We start with the highest-degree node, and enforce the policy that this node give priority to connect with the lowest-degree nodes, until its degree is exhausted. We then move on to the next highest-degree node. In this construction, nodes with high degrees are sparsely connected with each other. The third type of graph (termed PLRG-descending) is constructed in the reverse manner as the former - a higher priority is given for nodes with high degrees to connect with each other. Upon applying our decomposition technique to these graphs, we observe that both PLRG-ascending and PLRG-descending show a very different decomposition structure than the standard PLRG graph. At the first level of decomposition itself, these graphs break down into many connected components, with no existence of a giant component. This simple example illustrates that replicating the degree distribution of a graph is not sufficient to reproduce its decomposition characteristics.

	PLRG	PLRG-ascending	PLRG-descending
Level 0	50%	1%	22%
Level 1	88%	95%	92%
Level 2	90%	97%	95%
Level 3	91%	97%	96%
Level 4	90%	98%	95%

TABLE IX

PERCENTAGE OF NODES IN THE LARGEST CONNECTED COMPONENT IN THE FIRST FIVE LEVELS OF DECOMPOSITION

C. Decomposition with different values of α

Until now in our decomposition, we have used a fixed value of $\alpha = 0.01$, as the fraction of nodes to be removed in each stage of the decomposition. In Table X we present results from carrying out the decomposition for different values of α. Below, we present only the figures for the relative size of the giant component in the first level of decomposition, we have observed similar trends for lower levels. We note that, except for the Waxman graph, the size of the giant component decreases with an increase in α, and after a certain point, there no longer exists a "giant" component.

α	AS Graph	PLRG	GLP	Waxman
0.005	59%	57%	33%	99%
0.010	45%	50%	21%	99%
0.020	14%	38%	1%	98%
0.030	3%	29%	1%	97%
0.040	1%	17%	1%	96%
0.050	1%	6%	1%	94%

TABLE X

PERCENTAGE OF NODES IN THE LARGEST CONNECTED COMPONENT IN DECOMPOSITION LEVEL 1 WITH VARYING α